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Merging of a massive binary due to ejection of bound stars

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ABSTRACT
From the results of numerical scattering experiments and simulations of a massive black hole binary in spherically symmetric and shallow cores it has been deduced that most likely the shrinking process stalls due to loss-cone depletion before emission of gravitational radiation becomes important. Here we follow a different approach and focus on the population of stars which is bound to the binary and so far has not received much attention. With simple assumptions which should not be sensitive to initial conditions we derive a lower limit for the mass of stars which needs to be ejected by the binary in order to coalesce. We also compute this mass in dependence on the steepness of the density profile according to which the stars are distributed. Our results are not as pessimistic as earlier conclusions and actually suggest that the BHs merge.

Key words: black hole physics – galaxies: evolution – galaxies: interactions – galaxies: kinematics and dynamics

1 INTRODUCTION

The existence of a massive black hole binary (BHB) is a natural consequence of the assumptions that galaxies harbour a massive black hole (BH) in their center and that galaxies merge with each other. The evolution of the binary can be split up in roughly three phases: First both cores with the BHs in their center spiral inwards to each other due to dynamical friction. When the BHs bind to each other and form a hard binary on the parsec scale, the semimajor axis continues to decay due to slingshot ejection of stars. Eventually, in the third phase, emission of gravitational radiation dominates further shrinking until the BHs coalesce (Begelman et al. 1980). It is still a matter of debate whether the BHs coalesce or the shrinking stalls before entering the final phase because no stars are left to interact with the binary (i.e. loss-cone depletion). However, the origin of X- and Z-shaped radio galaxies is probably best explained by the coalescence of the two BHs (Rottmann 2001; Zier & Biermann 2002; Gopal-Krishna, et al. 2003; Zier 2005) and the observed number of these sources is in agreement with the merging rate derived for radio galaxies (Merritt & Ekers 2002). Numerical scattering experiments showed that the BHs can merge on scales of $\sim 10^{8-9}\,\mathrm{yr}$ if the loss-cone is always full (Quinlan 1996; Quinlan & Hernquist 1997; Zier & Biermann 2001; Milosavljević & Merritt 2001). However, for a flat core most authors estimate that the loss-cone gets depleted long before the binary can enter the third phase. Further hardening then depends on the rate at which the loss-cone is refilled, most probably by two-body scattering of stars. This implicates a time scale for the separation of the BHs to shrink to a distance where gravitational radiation becomes dominant which might exceed a Hubble time so that the binary basically stalls. The conclusions are essentially the same: though the results show that the BHs coalesce it is argued that in real galaxies the loss-cone is depleted before gravitational radiation dominates further shrinking. These conclusions are based on the assumption that at the time the binary becomes hard the density profile of the cusp is flat (i.e. with a power-law index of at most 2, usually 1 or even 0). This seems to have been confirmed by recent simulations of a binary in a constant density core (Berczik et al. 2005). As the authors say themselves the choice for the mass of the binary is unrealistically large compared to that of the galaxy and the initial conditions are quite unlikely, because the BHs were introduced symmetrically about the center of the galaxy instead of arriving there due to the evolution of the merger.

The initial conditions in these numerical experiments are idealised and cover only a small fraction of the parameter space. They are based on spherically symmetric profiles which have been derived from observations of elliptical galaxies. These profiles correspond to the central cluster before or after the merger when mass has already been ejected from the loss-cone and shifted from smaller to larger radii, resulting in flatter profiles. However, they do not match the central profiles *during* the merger. Also ignored are individual spins of both galaxies, which might stabilize the cluster against tidal disruption by the other BH beyond a hard binary, depending on the magnitude of the spins and their orientation relative to each other and relative to the orbital angular momentum of the merging galaxies. These param-

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eters have a strong influence on the merger itself and the morphology of the remnant, as has been shown by Toomre & Toomre (1972). When galaxies collide energy will be dissipated and angular momentum redistributed with some fractions compensating each other. Large amounts of mass will move on highly eccentric orbits (Rauch & Tremaine 1996) in a potential that is strongly non-spherically symmetric so that the angular momentum of a single particle is not conserved and matter with low angular momentum piles up in the center. The density of both cores will be increased considerably before they merge and the binary becomes hard (Barnes & Hernquist 1996). Each of the BHs will carry a stellar cusp as massive as the BH and therefore a stellar mass comparable to that of the binary will be concentrated in the central core once the cusps merge. Hence we expect that at the time the BHs form a hard binary, the surrounding density distribution will be much more compact with a steeper profile than in non-interacting galaxies, i.e. unlike the initial conditions used so far in numerical experiments. It is the fraction of this cusp which is bound to the binary to which our analyses applies.

Here we will study in a simple approach how compact such a cusp has to be and wich profile is required so that the binary can shrink to the third phase and the BHs eventually coalesce. Whereas numerical experiments were more concerned with unbound stars scattered off the binary, we will focus on stars which are bound by the BHs. Therefore our results will be less sensitive to initial conditions.

2 PRELIMINARIES

Being interested in the 2nd phase of the merger we assume that the BHs have bound to each other and are moving on Keplerian orbits. The origin is the center of mass of the binary and we define the mass ratio $q \equiv m_2/m_1 \leq 1$. The total and reduced mass are $M_{12} = m_1 + m_2$ and $\mu = m_1 m_2/M_{12}$, respectively. With a we denote the semimajor axis of the binary, i.e. of the orbit of the reduced mass. For circular orbits this is equal to the separation of the BHs. Thus the energy of the binary can be written as

$$E_{\rm bin} = -\frac{GM_{12}\mu}{2a}, \tag{1}$$

and the relative velocity between the BHs is

$$v_\mu = \sqrt{\frac{GM_{12}}{a}}, \tag{2}$$

which corresponds to the velocity of the reduced mass if it moves on circular orbits. The definition for the semimajor axis $a_{\rm h}$ where the binary becomes hard is not unique and changes in the literature (e.g. Heggie 1975; Hills 1975; Hut 1983; Quinlan 1996; Milosavljević & Merritt 2001). Some definitions are derived from the results of numerical experiments and are more phenomenological. Usually they are all based on comparing the velocity dispersion of the cluster (which itself is not so easy to define in an ongoing merger when the core is far from being relaxed) with the velocity of a component of the binary. For major mergers, i.e. large q, they all yield similar results for $a_{\rm h}$, which is found to be on the parsec scale for $M_1 \simeq 10^8\,M_\odot$.

We think the transition from the first to the second phase is best defined by the distance between the BHs when the stars are moving in the potential of both BHs, not just one, so that the binary and the star can be treated as a restricted three body problem. This transition from dynamical friction to slingshot ejection as dominating processes for the decay of the binary is smooth. Because we do not know an exact definition for $a_{\rm h}$ we instead scale it to the semimajor axis $a_{\rm g}$ at the end of the second phase, which is more clearly defined. An upper limit for this transition is if it still takes a Hubble time for the BHs to merge completely from $a = a_{\rm g}$ due to emission of gravitational radiation. For circular orbits this is (Peters 1964)

$$\begin{aligned} a_{\rm g} &= \left[\frac{256}{5}\,\frac{G^3\mu M_{12}^2}{c^5}\,t_{\rm g}\right]^{1/4} \\ &\approx \frac{1}{15}\left(\frac{M_1}{10^8 M_\odot}\right)^{3/4}\left(\frac{t_{\rm g}}{10^{10}\,{\rm yr}}\right)^{1/4}\,[q(1+q)]^{\frac{1}{4}}\ {\rm pc}. \end{aligned} \tag{3}$$

In previous publications the ratio of the semimajor axes where the transitions between the phases occur, i.e. $\eta \equiv a_{\rm h}/a_{\rm g}$, has been found to be in the range of $20-100$. Applied to Eq. (3) this yields $a_{\rm h}$ to be in the range $1-7\,$pc, in agreement with $a_{\rm h}$ being on the parsec scale. Because this ratio does not seem to depend sensitively on the initial conditions and is quite robust we will use it in the following to compute the location of $a_{\rm h}$. Note that $a_{\rm h}$ and $a_{\rm g}$ scale differently with the mass of the BHs and the velocity dispersion of the cluster. Consequently the ratio η will be a function of these quantities. Because we are considering a range of fixed values for η, this does not affect our following analysis and might become important only if systems with very different BH masses and velocity dispersions are considered.

For bound stars it makes only a small difference whether the stellar distribution and potential is spherical, axisymmetric or triaxial. For simplicity we consider a spherical density distribution which in the inner region follows a power-law, $\rho = \rho_0 (r/r_0)^{-\gamma}$. With the total mass of this cluster ($M_{\rm c}$) being distributed between the inner and the cluster radius, $r_{\rm i}$ and $r_{\rm c}$ respectively, the mass distribution is

$$M(r) = M_{\rm c}\begin{cases} \dfrac{r^{3-\gamma} - r_{\rm i}^{3-\gamma}}{r_{\rm c}^{3-\gamma} - r_{\rm i}^{3-\gamma}}, & \gamma \neq 3 \\ \dfrac{\ln(r/r_{\rm i})}{\ln(r_{\rm c}/r_{\rm i})}, & \gamma = 3. \end{cases} \tag{4}$$

To avoid a singularity at $r = 0$ we cut off the distribution at the inner radius $r_{\rm i}$. The circular velocity is $v_{\rm circ}^2 = r\,d\Phi/dr = GM(r)/r$. A star with this velocity is bound to a BH with mass M_{12} at the center of the cluster if the velocity is less than the escape velocity $v_{\rm esc} = \sqrt{2GM_{12}/r}$. Using the circular velocity as the typical velocity (i.e. the velocity at the maximum of an isothermal sphere with a Maxwellian velocity distribution, same as $\sqrt{2}$-times the velocity dispersion of this distribution) we obtain from $v_{\rm circ} \leq v_{\rm esc}$ the relation $M(r) \leq 2M_{12}$. Thus a mass of about $2M_{12}$ of the cluster is bound to the binary. We expect a large fraction of this mass to be in the loss-cone.

3 REQUIRED MASS AND ITS DISTRIBUTION

In this section we will derive limits for the mass which is required to be ejected and how steep the mass distribution has to be so that the BHs can merge. The inner region where the stars are bound to the binary is dominated by the potential

of the BHs. Approximating the potential of the binary to first order with a point potential of the mass M_{12} located at the cluster's center introduces only minor deviations whith a maximum of a factor less than 2 for $q = 1$. Before a star becomes ejected via the slingshot mechanism its binding energy is about

$$E_{*,\mathrm{i}} = -(1-\epsilon)\frac{GM_{12}m_*}{2r_-}, \tag{5}$$

where r_- denotes the pericenter and $\epsilon < 1$ the eccentricity of the orbit. In this expression we neglected the potential of the cluster itself, whose mass amounts up to twice the mass of the binary, as we have shown in the previous section. The influence of the potential of the cluster could be explored in self-consistent calculations using a potential generated by the cluster until its mass drops below that of the binary. We do not expect the cluster potential to change the basic results obtained in the present work and leave these calculations to a subsequent paper. The final energy of the star after its ejection, $E_{*,\mathrm{f}}$, is zero or more. Scaling it to the initial energy with $\epsilon = 0$ we can write it as $E_{*,\mathrm{f}} = \kappa GM_{12}m_*/2r_-$, with κ being the scaling factor. Independent of the density profile Quinlan (1996) finds that the dominant contribution to the hardening of the binary comes from stars whose closest approach to both BHs is about the semimajor axis a. Hence we replace r_- with a so that the energy change of the star $E_{*,\mathrm{f}} - E_{*,\mathrm{i}}$ can be written as

$$\Delta E_* = (1-\epsilon+\kappa)\frac{GM_{12}m_*}{2a} \equiv k\,\frac{GM_{12}m_*}{2a}, \tag{6}$$

with k being defined by the last equality. From scattering experiments it follows that $\kappa \approx (3/2)^2 m_2/M_{12}$ (Quinlan 1996) or $\sim \mu/M_{12}$ (Saslaw et al. 1974). Thus for circular orbits ($\epsilon = 0$) we find k in the range $1 \lesssim k \lesssim 2$, or as large as 3.2 according to Yu (2002). Even for highly eccentric orbits ($\epsilon \sim 0.7$) the energy change will be of the order of one.

3.1 The ejected mass

When a star is ejected it extracts the amount of energy given in Eq. (6) from the binary. In the limit $m_* \ll m_2$ we can replace m_* with dm and using Eq. (1) we can write $dE_{\mathrm{bin}} = \Delta E_*$ as

$$\frac{da}{a} = -k\frac{dm}{\mu}, \tag{7}$$

giving an expression for the shrinking of the binary da due to the ejection of a mass dm. Stars are ejected all the time as long as the loss-cone is not depleted and so the binary hardens continuously. Integration from a_{g} to a_{h} yields the mass that has to be ejected by the binary in order to enter the last phase and we obtain

$$m_{\mathrm{ej}} = \frac{\mu}{k}\ln\frac{a_{\mathrm{h}}}{a_{\mathrm{g}}}. \tag{8}$$

This is only slightly more than M_{12} for $q = k = 1$ and $\eta = 100$ and therefore less than the mass bound to the binary. In deriving this expression we assumed that the stars are moving on orbits with pericenters which are equal to the current semimajor axis of the binary. However, stars interior to the binary will at least be disturbed, if not ejected, once the binary has become hard. These stars are more tightly bound in the potential of the BHs and have to gain energy on the expense of the binary in order to be shifted to orbits with a as pericenter. This issue will be explored in detail in the next section which shows that the mass obtained in Eq. (8) is sufficient for a merger if it is distributed with a power-law index $\gamma = 3$.

To define the mass ejection rate Quinlan (1996) introduced a similar expression as Eq. (7) and used as mass scaling factor M_{12} without further justification. This is plausible for large q because in this case it can be expected that the binary has to eject a mass of about its own. This was confirmed by his results which are essentially in agreement with ours. However, in the limit $m_2 \to m_*$ it does not seem to be plausible that a total mass of about M_{12} is required. The energy of the binary is that of the reduced mass orbiting in the fixed potential of the point mass M_{12} with an angular momentum of $L_{\mathrm{bin}} = \mu a v_\mu$. Therefore a mass of about μ, as derived in the above equation, seems to be more natural. In order to explain mass deficits in galaxy cores Milosavljević et al. (2002) find that a scaling factor of m_2 instead of M_{12} matches the ejected mass better. This dependency on q is very similar to that in our expression, supporting our approach of focussing on the bound population of stars. Because we did not make use of any further assumptions in the derivation of Eq. (8) it should best describe the true ejected mass.

3.2 The distribution of the ejected mass

According to Milosavljević & Merritt (2001) the results of their simulations indicate that soon after the binary becomes hard the loss-cone becomes depleted in little more than the local crossing time of the cusp. If we assume that all the mass of Eq. (8) is ejected instantaneously at a_{h}, i.e. that the density is distributed according to a Dirac-delta distribution that is non-vanishing at $r = a_{\mathrm{h}}$, the energy change due to ejection of this mass is $\Delta E_{\mathrm{ej}} = (GM_{12}\mu/2a_{\mathrm{h}})\ln(a_{\mathrm{h}}/a_{\mathrm{g}})$ (Eq. [6]). Equating this with the change in energy of the binary in Eq. (1) if the semimajor axis shrinks from a_{h} to the final semimajor axis a_{f}

$$\Delta E_{\mathrm{bin}} = \frac{GM_{12}\mu}{2a_{\mathrm{h}}}\left(\frac{a_{\mathrm{h}}}{a_{\mathrm{f}}} - 1\right) \tag{9}$$

and solving for their ratio we obtain

$$\frac{a_{\mathrm{h}}}{a_{\mathrm{f}}} = 1 + \ln(a_{\mathrm{h}}/a_{\mathrm{g}}), \tag{10}$$

which is only about 5.6, 4.9 and 4.0 for $a_{\mathrm{h}}/a_{\mathrm{g}} = 100$, 50 and 20 respectively. However, stars at radii smaller than a_{h} are also bound deeper in the potential of the binary. Before the above derivation for a_{f} applies the BHs have to transfer sufficient energy to these stars so that they move on orbits with a radius of a_{h}. In the point potential the binding energy of a mass element $dm = 4\pi r^2\rho(r)\,dr$ in a distance r is $dE = GM_{12}dm/2r$. The integration over the cusp's profile between a_{g} and a_{h} gives the binding energy of the cusp in this potential and corresponds to the energy which the binary loses when it ejects these stars. This negelects the potential of the cusp itself, and because the final energies of the stars are probably more than zero this is actually a lower limit for the energy loss of the binary. Using the density profile $\rho = \rho_0(r/r_0)^{-\gamma}$, and with the help of Eq. (4),

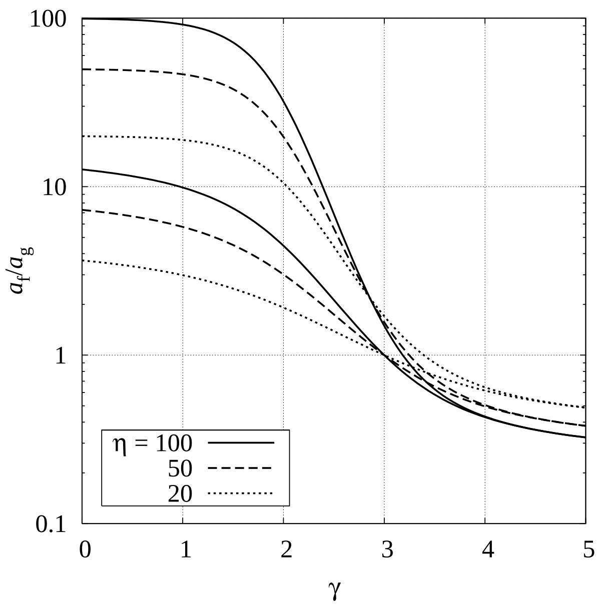

Figure 1. The final semimajor axis in units of $a_{\rm g}$ as function of the exponent γ. Solid, dashed and dotted lines show the distribution for different ratios $\eta = a_{\rm h}/a_{\rm g}$. For $\gamma \lesssim 4$ they split up in a lower ($\lambda = 1$) and upper ($\lambda = 10$) branch.

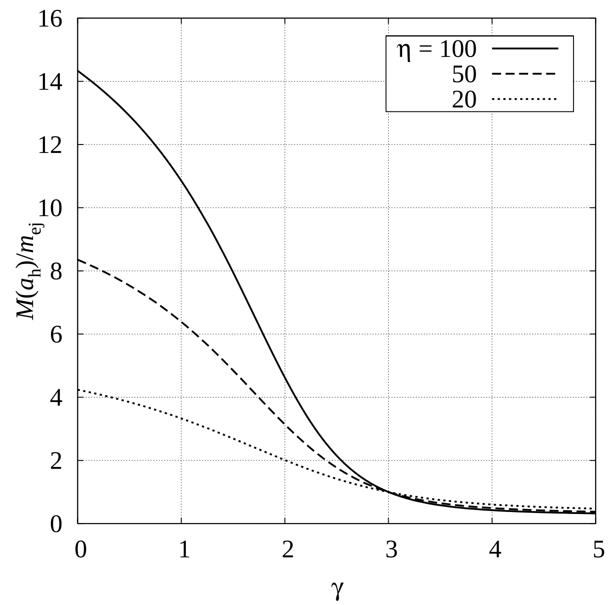

Figure 2. The mass distributed between $a_{\rm g}$ and $a_{\rm h}$ which is required to allow the BHs to shrink to $a_{\rm g}$ in units of $m_{\rm ej}$ as function of the exponent γ. ($k = 1$).

the integration yields

$$\Delta E = \frac{GM_{21}M_{\rm c}}{2}\begin{cases}\dfrac{1}{r_{\rm c}-r_{\rm i}}\ln(a_{\rm h}/a_{\rm g}) & \gamma = 2\\ \left(\dfrac{1}{a_{\rm g}}-\dfrac{1}{a_{\rm h}}\right)\left(\ln\dfrac{r_{\rm c}}{r_{\rm i}}\right)^{-1} & \gamma = 3\\ \dfrac{3-\gamma}{2-\gamma}\,\dfrac{a_{\rm h}^{2-\gamma}-a_{\rm g}^{2-\gamma}}{r_{\rm c}^{3-\gamma}-r_{\rm i}^{3-\gamma}} & \text{else.}\end{cases} \quad (11)$$

We assume that the lower limit for the mass which is required to allow the BHs to merge, found in Eq. (8), is the cluster mass $M_{\rm c}$ which is distributed between $r_{\rm i}$ and $r_{\rm c}$ according to the above power law. If the binary ejects the stars from the region $a_{\rm g} \le r \le a_{\rm h}$ we can compute the final semimajor axis of the binary in the same way as above, i.e. equating Eqs. (9) and (11), and find:

$$\frac{a_{\rm h}}{a_{\rm f}} = \begin{cases}1+\dfrac{1}{k\lambda}\dfrac{\zeta}{\zeta-1}(\ln\eta)^2 & \gamma = 2\\ 1+\dfrac{\eta-1}{k}\dfrac{\ln\eta}{\ln\zeta} & \gamma = 3\\ 1+\dfrac{\lambda^{\gamma-3}}{k}\,\dfrac{3-\gamma}{2-\gamma}\,\dfrac{1-\eta^{\gamma-2}}{1-\zeta^{\gamma-3}}\ln\eta & \text{else.}\end{cases} \quad (12)$$

Here we have defined the following ratios: $\eta = a_{\rm h}/a_{\rm g} > 1$ as before, $\zeta \equiv r_{\rm c}/r_{\rm i} > 1$ and $\lambda \equiv r_{\rm c}/a_{\rm h} \ge 1$. If we allow $r_{\rm i} \le a_{\rm g}$, so that the cluster extends to radii smaller than $a_{\rm g}$, the fraction of mass between these two radii is not included in the ejected mass since we used $a_{\rm g}$ as lower limit in the integration (Eq. [11]). The steeper the distribution, the larger is this mass fraction, and hence the less the binary will shrink so that $a_{\rm f}/a_{\rm g}$ increases with γ, if γ exceeds a certain value. However, these solutions are unphysical and will not be considered. Therefore, setting $r_{\rm i} = a_{\rm g}$, we can substitute $\zeta = \lambda\eta$. In Fig. 1 we plotted the result for $k = 1$ as ratio $a_{\rm f}/a_{\rm g} = \eta\, a_{\rm f}/a_{\rm h}$ in dependence on the exponent γ. As expected, the steeper the cusp the closer the binary shrinks to $a_{\rm g}$. In the range $\gamma \lesssim 4$ the solutions split into a lower ($\lambda = 1$) and upper ($\lambda = 10$) branch. In case of $\lambda = 1$ the mass of Eq. (8) is distributed between $a_{\rm g}$ and $a_{\rm h}$, which coincide with $r_{\rm i}$ and $r_{\rm c}$ respectively. For a uniform density distribution ($\gamma = 0$) the final semimajor axis is smaller by a factor of about 1.4 than for the case of the Dirac-delta distribution which yields $a_{\rm f}/a_{\rm g} \approx 17.8$, 10.2 and 5.0 for $\eta = 100$, 50 and 20 respectively, see Eq. (10). The decrease of the final semimajor axis with increasing γ is steeper for large η. If we write in Eq. (7) $dm = 4\pi r^2\rho(r)dr$ and solve for the density, we obtain $\rho(r) = \mu/4\pi k r^3$, i.e. the power-law for which $m_{\rm ej}$ is just enough to allow the BHs to merge ($a_{\rm f}/a_{\rm g} = 1$ at $\gamma = 3$). For a distribution as steep or steeper than $\rho \propto r^{-3}$ the ejected mass is bound deeply enough in the potential of the binary prior to its ejection for all ratios η in order to allow the BHs to shrink to the distance where gravitational radiation begins to dominate the further decay. Hence for sufficiently steep and compact distributions the BHs coalesce.

If $\lambda = 10$ the mass $m_{\rm ej}$ is distributed in a larger sphere with an outer radius 10 times as large as $a_{\rm h}$. The larger λ is the closer the ratio $a_{\rm f}/a_{\rm g}$ approaches η at $\gamma = 0$ because less mass is available for ejection at radii $\le a_{\rm h}$, and the less the binary will shrink (upper branches in Fig. 1). The dependency on the exponent is basically the same as for $\lambda = 1$ and for sufficiently steep distributions the ejected mass fraction removes sufficient energy so that the BHs can coalesce. At $\gamma = 3$ we can approximate $a_{\rm f}/a_{\rm g}$ with $1 + \ln\lambda/\ln\eta$, a function which increases slowly with λ. Even for the cluster extending as far as $r_{\rm c} = \eta a_{\rm h}$ only about twice as much mass is needed for the BHs to coalesce as compared to $\lambda = 1$.

To obtain the mass required for coalescence of the BHs for other exponents than $\gamma = 3$, where exactly $m_{\rm ej}$ is needed (see Fig. 1), we can proceed as follows. In Eq. (12) we assume the cluster mass to be distributed between $a_{\rm g}$ and $a_{\rm h}$, i.e. $\lambda = 1$ and $\zeta = \eta$. Scaling the required mass to $m_{\rm ej}$ with a

factor $\chi \equiv M(a_{\rm h})/m_{\rm ej}$ we just need to replace $1/k$ with χ/k in Eq. (12) and solve $a_{\rm h}/a_{\rm f} = \eta$ for χ. This yields

$$\frac{M(a_{\rm h})}{m_{\rm ej}} = k\,\frac{2-\gamma}{3-\gamma}\,\frac{1-\eta^{\gamma-3}}{1-\eta^{\gamma-2}}\,\frac{\eta-1}{\ln\eta}. \tag{13}$$

Plotting this ratio as function of the exponent for $k = 1$ again shows that for flat distributions ($\gamma \lesssim 2$) much more mass is required than for steeper cusps, at least for $\eta \gtrsim 50$ (Fig. 2). For $\gamma \approx 2.5$ this is less than $2\,m_{\rm ej}$ for all η, confirming our conclusion that the binary is likely to merge in compact cusps which are steeper than $\gamma = 2$.

Steep density distributions with $\gamma \gtrsim 2$ have not been explored in numerical simulations or scattering experiments which used cusps at most as steep as $\gamma = 2$. Considering the population of stars which is bound to the binary we obtain quite different results which show that the BHs can actually merge if the cusp is steep enough.

4 DISCUSSION AND CONCLUSIONS

Although numerical simulations showed that the BHs in a hard binary coalesce after about 10^{7-8} yr if the loss-cone is full (e.g., Quinlan 1996; Milosavljević & Merritt 2001; Zier & Biermann 2001) most authors argued that the loss-cone becomes depleted and the binary stalls. This conclusion is based on the assumption of a spherically symmetric shallow density profile and has been confirmed recently by Berczik et al. (2005), using initial conditions which are in favour of a stalled binary. The choice of a flat central density distribution is based on profiles derived from observations of elliptical galaxies, i.e. galaxies before or (more probably) after a merger, when mass has been redistributed from the inner to the outer parts of the cluster, resulting in a flatter profile. However, *during* the merger the profile might be quite different and much steeper. Simulations by Toomre & Toomre (1972) support this conjecture and Barnes & Hernquist (1996) showed that both cores accumulate a big amount of mass before they merge and the binary becomes hard. Each of the BHs will carry a stellar cusp with a mass of about its own.

Hence in this Letter we suggest that by the time the BHs become hard, mass and angular momentum have been redistributed and energy dissipated so that the central cusp is more massive and has a steeper density profile than that deduced from observations. Naturally this is a transient distribution and unlikely to be observed because of its short lifetime. Because a mass of about $2\,M_{12}$ will bind to the binary, a large fraction of which will probably be contained in the loss-cone, we focused on this population, unlike numerical experiments which so far have focussed on scattered unbound stars. By computing the binding energy of a stellar distribution in the potential of a binary we could show that the ejection of this population can play a decisive role for a successful merger. From the energy extracted from the binary by these stars due to sling-shot ejection we derived a new expression for the mass which is required to be ejected so that the BHs coalesce (Eq. [8]). While previous work assumed this mass to be proportional to the mass of the binary M_{12} due to the choice of the scaling factor, we find it to be proportional to the reduced mass. This seems to be more plausible, especially in the limit of a small secondary BH, and is also in good agreement with fits to the mass deficit derived from observations (Milosavljević et al. 2002). Because the mass bound to the binary is more than twice as much as the mass required for coalescence we conclude that the fraction of bound stars in the loss-cone contributes at least a significant fraction to the coalescence and might be enough to enable merging until $a = a_{\rm g}$ on its own. We assumed that this mass is moving on circular orbits and approximated the potential of the binary by that of a point mass. In Figs. 1 and 2 we showed that the ejection of mass, which is distributed in the potential of the binary according to a flat profile ($\gamma \lesssim 2$) as in previous papers, does not allow the BHs to merge. Either the binary stalls or a very large amount of mass is required. However, these figures also show that for steeper distributions the ejected mass is indeed sufficient to remove enough energy from the binary so that the BHs can coalesce.

If we allow the stars to move on eccentric orbits using their former radius as pericenter, the binding energy will decrease while the required ejected mass will increase. However, this will be more than compensated by the non-vanishing velocity of ejected stars at infinity, which we assumed to be 0. Comparing Eq. (6) with Yu (2002) we find $k = 3.2$, including elliptic orbits. Elliptic orbits allow the matter to be distributed less steeply than what is suggested by our results and therefore relax the conditions for a complete merger. We also assumed the BHs to move on circular orbits. If they are elliptic gravitational radiation dominates the shrinking earlier, shifting $a_{\rm g}$ to larger radii and consequently reducing the mass required for ejection. Therefore our results in fact are quite conservative and we conclude that the BHs coalesce in most cases, contrary to previous results which were based on flat cores. Our analyses can also be applied to dark matter. If there is not sufficient baryonic matter for a merger and the BHs still merge, our approach allows some conclusions to be drawn about the central distribution of dark matter.

Simulations of merging BHs in a triaxial potential (Yu 2002; Holley-Bockelmann et al. 2002; Holley-Bockelmann & Sigurdsson 2006; Berczik et al. 2006) strongly support the conclusion that the BHs merge completely in less than a Hubble time. Due to the triaxiality of the potential, the central regions remain populated. The flow of orbits through phase space into the loss-cone is large enough to keep it always filled. An unfilled loss-cone is only found for shallow spherically symmetric cores (Yu 2002). This is an idealized distribution which, if at all, applies only to the cluster after the merger has been completed, but not to an ongoing merger. Thus, additionally to the population bound to the binary that we were investigating here, simulations of a triaxial potential show that the loss-cone remains filled during the merger, supplying the binary with an almost unlimited amount of stars that can be scattered off. Even in the case that the bound stars alone are not enough to allow the BHs to merge, together with the scattered stars from the loss-cone a stalled binary should be most unlikely.

In a future paper (in preparation) we will discuss this approach and its context in more detail.

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REFERENCES

Barnes J. E., Hernquist L., 1996, ApJ, 471, 115
Begelman M. C., Blandford R. D., Rees M. J., 1980, Nat., 287, 307
Berczik P., Merritt D., Spurzem R., 2005, ApJ, 633, 680, astro-ph/0507260
Berczik P., Merritt D., Spurzem R., Bischof H.-P., 2006, ApJ, 642, L21, astro-ph/0601698
Gopal-Krishna, Biermann P. L., Wiita P. J., 2003, ApJ, 594, L103, astro-ph/0308059
Heggie D. C., 1975, MNRAS, 173, 729
Hills J. G., 1975, AJ, 80, 809
Holley-Bockelmann K., Mihos J. C., Sigurdsson S., Hernquist L., Norman C., 2002, ApJ, 567, 817, astro-ph/0111029
Holley-Bockelmann K., Sigurdsson S., 2006, astro-ph/0601520
Hut P., 1983, ApJ, 272, L29
Merritt D., Ekers R. D., 2002, Science, 297, 1310, astro-ph/0208001
Milosavljević M. ., Merritt D., 2001, ApJ, 563, 34, astro-ph/0103350
Milosavljević M., Merritt D., Rest A., van den Bosch F. C., 2002, MNRAS, 331, L51, astro-ph/0110185
Peters P. C., 1964, Phys. Rev. B, 136, 1224
Quinlan G. D., 1996, New Astronomy, 1, 35, astro-ph/9601092
Quinlan G. D., Hernquist L., 1997, New Astronomy, 2, 533, astro-ph/9706298
Rauch K. P., Tremaine S., 1996, New Astronomy, 1, 149, astro-ph/9603018
Rottmann H., 2001, PhD thesis, University of Bonn
Saslaw W. C., Valtonen M. J., Aarseth S. J., 1974, ApJ, 190, 253
Toomre A., Toomre J., 1972, ApJ, 178, 623
Yu Q., 2002, MNRAS, 331, 935, astro-ph/0109530
Zier C., 2005, MNRAS, 364, 583, astro-ph/0507129
Zier C., Biermann P. L., 2001, A&A, 377, 23, astro-ph/0106419
Zier C., Biermann P. L., 2002, A&A, 396, 91, astro-ph/0203359